

Pjesenje

1. (i) ~~$\frac{\partial f}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x}$~~

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x}$$

$$\frac{\partial f}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x, y+\Delta y) - f(x, y)}{\Delta y}$$

(ii) $\frac{\partial f}{\partial x} = 3x^2 y^2$; $\frac{\partial f}{\partial y} = 2x^3 y$

(iii) $\frac{\partial f}{\partial x}(1, 2) = 3 \cdot 1^2 \cdot 2^2 = \underline{12}$; $\frac{\partial f}{\partial y}(-1, 2) = 2 \cdot (-1)^3 \cdot 2 = \underline{-4}$

2. (i) $f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0) \Delta x + \frac{\partial f}{\partial y}(x_0, y_0) \Delta y$
Može i:

$$f(x, y) \approx f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$$

(ii) $\Delta f(x_0, y_0) = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)$

$$\Delta f(x_0, y_0) \approx \frac{\partial f}{\partial x}(x_0, y_0) \Delta x + \frac{\partial f}{\partial y}(x_0, y_0) \Delta y$$

(iii) $df(x_0, y_0) = \frac{\partial f}{\partial x}(x_0, y_0) dx + \frac{\partial f}{\partial y}(x_0, y_0) dy$

(3)

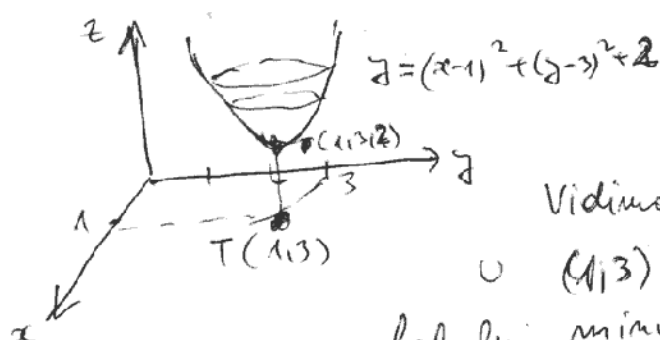
3. (i) ~~SS~~ Ako je $u(x_0, y_0)$ lokalni ekstrem
onda je $\frac{df}{dx}(x_0, y_0) = 0$ i $\frac{df}{dy}(x_0, y_0) = 0$.

(ii) $\frac{df}{dx} = 2x - 2$; $\frac{df}{dy} = 2y - 6$

Kritične tačke su rješenja sistema $\frac{df}{dx} = 0$ i $\frac{df}{dy} = 0$,

g. $2x - 2 = 0$ i $2y - 6 = 0$ g. $x = 1$ i $y = 3$
Tačka $T(1, 3)$ je jedina kritična tačka.

(iii) Vpiedi $f(x, y) = (x-1)^2 + (y-3)^2 + 2$: Zato je to:



Vidimo da je
 $u(1, 3)$ ~~minimum~~
lokalni minimum i da je
vpiedost minimuma 2.

4. (i) $\iint_D f(x, y) dx dy$ za pozitivnu funkciju f ima
zastupanje obujma tijela iznad područja D i
gore funkcije f . Pozegna vlika !!

(ii) $\iint_D f(x, y) dx dy = \int_a^b \left[\int_{\varphi(x)}^{\psi(x)} f(x, y) dy \right] dx$

ako je D zadano : $a \leq x \leq b$
 $\varphi(x) \leq y \leq \psi(x)$

Pozegna vlika!

(4)

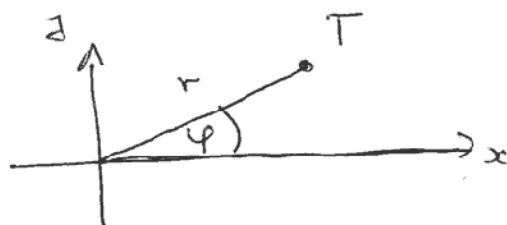
Tu je D : $0 \leq x \leq a$
 $0 \leq y \leq b - \frac{b}{a}x$

jer je jednačina dijametrala $\frac{x}{a} + \frac{y}{b} = 1$ tj. $y = b - \frac{b}{a}x$

Zato je

$$\begin{aligned} \iint_D x \, dx \, dy &= \int_0^a \left[\int_{b - \frac{b}{a}x}^b x \, dy \right] dx \\ &= \int_0^a \left(x y \Big|_{y=b - \frac{b}{a}x}^{y=b} \right) dx = \int_0^a x \left(b - b + \frac{b}{a}x \right) dx \\ &= \int_0^a \frac{b}{a} x^2 dx = \frac{b}{a} \frac{x^3}{3} \Big|_0^a = \frac{b}{a} \cdot \frac{a^3}{3} = \frac{a^2 b}{3}. \end{aligned}$$

5. (i)

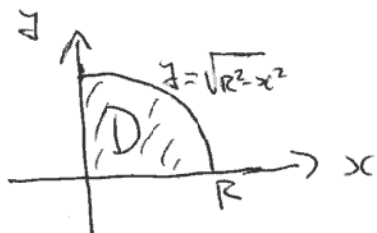


(ii)



$$\begin{aligned} 0 < \varphi < \frac{\pi}{2} \\ 0 < r < +\infty \end{aligned}$$

(iii)



D je četvrtina kruga poluprečnika R u I kvadrantu. U polarnim koordinatama to je:

$$0 \leq \varphi \leq \frac{\pi}{2} \text{ i } 0 < r \leq R. \text{ Zato je}$$

$$(42 \quad \sqrt{x^2 + y^2} = r)$$

$$\begin{aligned} \iint_D \sqrt{x^2 + y^2} \, dA &= \int_0^{\frac{\pi}{2}} \left[\int_0^R r \cdot r \, dr \right] d\varphi \\ &= \int_0^{\frac{\pi}{2}} \left(\frac{r^3}{3} \Big|_0^R \right) d\varphi = \int_0^{\frac{\pi}{2}} \frac{R^3}{3} d\varphi = \frac{R^3}{3} \varphi \Big|_0^{\frac{\pi}{2}} \\ &= \frac{R^3}{3} \cdot \frac{\pi}{2} = \frac{R^3 \pi}{6}. \end{aligned}$$